OPTIMAL TRANSMISSION EXPANSION PLANNING WITH GENERATOR/LOAD MODELS AND FREQUENCY CONTROLS USING DIFFERENTIAL EVOLUTION ALGORITHM

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Abstract- Cost-effective transmission expansion planning (TEP) is a major challenge of the power system optimization problems. The main purpose of TEP problem is to determine the optimal expansion plan of the electrical transmission system. Furthermore, TEP should specify the new circuits that have to be added to an existing network to guarantee adequate operation for a specified planning horizon. Usually, TEP can be categorized as static or dynamic (multistage) planning according to the study period. Static planning involves a single planning horizon, whereas dynamic planning is a derived generalization that considers the separation of the planning horizon into multiple stages. In the past few years, various evolutionary algorithms, heuristic or metaheuristic algorithms are being applied for expansion planning. Differential evolutionary algorithm (DEA) has been used to solve a wide range of power system problems such as static TEP, short-term scheduling of hydrothermal power systems, power system planning etc. In a number of cases, DEA is proven to be reliable providing optimal solutions with acceptable computational effort. In this paper DEA is proposed to solve the TEP, based on AC-OPF problem, using AC model. In practical scenario, the load/generator is not in a single type. In this paper TEP is approaches to be carried out with generator model and various load models. In order to optimize the transmission network topology by minimizing the objective function, this paper deals with selection of new circuits which should be added to the existing transmission network. It is, however subjected to operating conditions for generating units and transmission network.

Keywords- AC-OPF, Differential Evolution Algorithm, Frequency Control, Generator Model, Load Models, Transmission Expansion Planning.

I. INTRODUCTION

The objectives of transmission expansion planning are based on existing systems, future load, generation scenarios, available right-of-ways, cost of line etc. The TEP is an important part of power system planning. It consists of determining the optimal expansion plan of transmission network such that the total cost of new constructed transmission lines to be minimize, while satisfying the operational constraints of the power system. TEP can be divided into two types 1) static TEP and 2) dynamic TEP. Static TEP performs all the expansions in a single stage of planning horizon. While dynamic TEP decides when, where and the number of new circuits to be installed to meet the growing electricity demand in an optimal way. The heuristic and metaheuristic techniques prove to be very well suited for many optimization problems. Many techniques such as genetic algorithm (GA), particle swarm optimization (PSO), simulated annealing (SA), tabu search (TS) etc. have been proposed and tested.

Recently an accurate AC network modeling has been proposed [3]. In the first phase use of the AC model is found incipient. Later there are a few technical literatures on the subject [3, 7, and 8]. Owing to the large-scale nature of a transmission system and its complexities, TEP has always been a complex and non-convex optimization problem. For better power systems utilization, developing a TEP model for the operating conditions is therefore desirable. Therefore reactive power sources are desire for increasing power transfer, improving power factor, reducing real power losses and maintaining voltage profile in a permissible range.

A differential evolution algorithm (DEA) introduced by Storn and Price in 1995 [9] is an evolutionary computation method. DEA developed is a reliable and versatile function optimizer readily applicable to a wide range of optimization problems [10]. DEA uses rather greedy selection and less stochastic approach to solve optimization problems than other classical evolutionary algorithms. There are also a number of significant advantages when using DEA, which were summarized by Price in [11] are (i) Ability to find the true global minimum regardless of the initial parameter values, (ii) Fast and simple with regard to application and modification, (iii) Requires few control parameters, (iv) Parallel processing nature and fast convergence, (v) Capable of providing multiple solutions in a single run, (vi) Effective on integer, discrete and mixed parameter optimization and (vii) Ability to find the optimal solution for a nonlinear constrained optimization problem with penalty functions etc. TEP has been studied on the standard test systems Garver – 6 bus system.
II. FREQUENCY CONTROLS AND MODELS

A. Generator Models
Under normal conditions, system frequency is maintained constant and generators are operated at a scheduled voltage and output. When system load changes, however, output of generators is varied by instruction from the Automatic Load Frequency Control (AFC). Governor setting is changed so that operation returns to a point on the governor load-speed curve corresponding to system operating frequency. When the system is disturbed by loss of generation or tie line support, the governor restores balance automatically, while generator terminal voltage is kept to a reference voltage within the limits of the exciter rating, and some generators are controlled automatically by their reactive power output.

Generator real power output is adjusted by the static response of the prime mover. This may be expressed as

\[ P_{g} = P_{gen} - \frac{P_{R}}{R} \Delta f \]  \hspace{1cm} (1)

and

\[ P_{g}^{min} \leq P_{g} \leq P_{g}^{max} \]  \hspace{1cm} (2)

Where

- \( P_{g} \) - is the real power output of the generator at \( i^{th} \) bus
- \( P_{gen} \) - is the scheduled power output of the generator at \( i^{th} \) bus
- \( P_{R} \) - is the rated output of the generator
- \( R \) - is the speed regulation in per unit

B. Frequency Controls
Automatic Load-frequency Control (AFC): Frequency fluctuations due to changes in load are monitored continuously, and the frequency is maintained constant by using a governor motor or limit to control generator output. AFC does this automatically and are (i) Flat Frequency control (FFC), and/or (ii) Flat Tie-line Control (FTC), and/or flat (iii) Flat Tie-line frequency Bias Control (TBC) on interconnected systems.

Flat Frequency control (FFC): In response to changes in system frequency, the power outputs of generators within a prescribed area are automatically regulated to maintain scheduled system frequency. Since the system frequency remains constant (i.e., \( \Delta f = 0 \), \( \Delta f \) is replaced by the new variable \( P_{RQ} \).

Power output for the regulating generators is

\[ P_{g} = P_{gen} + \alpha_{i} P_{RQ} \] \hspace{1cm} (3)

Where

- \( P_{g} \) - is the real power output of the generator at \( i^{th} \) bus
- \( P_{gen} \) - is the scheduled power generation at \( i^{th} \) bus
- \( P_{RQ} \) - is the supply insufficiency in a given area,
- \( \alpha_{i} \) - is the load distribution factor of \( i^{th} \) generator such that \( \sum \alpha_{i} = 0 \)

C. Load Models
Load models are traditionally classified into two broad categories, static models and dynamic models. Static Load Models: These models express the active and reactive powers, at any instant of time, as a function of the bus voltage magnitude and frequency. Static load models are used in both static and dynamic load components.

Dynamic Load Model: A Dynamic load model expresses the active and reactive powers at any instant of time as functions of the voltage magnitude and frequency. Studies of inter area oscillations, voltage stability, and long term stability often require load dynamic to be modeled.

Different types of Static and Dynamic Load Models:
The following different types of static and dynamic load models
a. Constant impedance load model is a static load model where the power varies directly with the square of the voltage magnitude. It may also be called a constant admittance load model.
b. Constant current load model is a static load model where the power varies directly with the voltage magnitude.
c. Constant power load model is a static load model where the power does not vary with changes in voltage magnitude. It may also be called constant MVA load model.

The static load model that represents the power relationship to voltage magnitude as a polynomial equation, usually in the following form:

\[ P_{a} = P_{ave} (1 + C_{f} \Delta f) \left[ K_{g} + K_{P} \frac{V}{V_{0}} + K_{Q} \left( \frac{V}{V_{0}} \right)^{2} \right] \] \hspace{1cm} (4)

\[ Q_{a} = Q_{ave} (1 + C_{f} \Delta f) \left[ K_{g} + K_{P} \frac{V}{V_{0}} + K_{Q} \left( \frac{V}{V_{0}} \right)^{2} \right] \] \hspace{1cm} (5)

where

- \( P_{a} \) - active power demand at \( i^{th} \) bus
- \( Q_{a} \) - reactive power demand at \( i^{th} \) bus
- \( P_{ave} \) - active power consumptions at rated voltage, \( V_{0} \) at \( i^{th} \) bus
- \( Q_{ave} \) - reactive power consumptions at rated voltage, \( V_{0} \) at \( i^{th} \) bus
- \( C_{f}, C_{P}, C_{Q} \) - constant of frequency characteristics of load
- \( V \) - supply voltage
- \( V_{0} \) - rated voltage
III. TEP PROBLEM FORMULATION

TEP problem is usually refers as a static transmission model. Generally, the objective of fitness function is to find optimal solution, measure performance of candidate solutions and check for violation of the planning problem constraints. Fitness function of the static TEP problem is basically a combination between objective function and penalty functions. The purpose of applying penalty functions to the fitness function is to represent violations of equality and inequality constraints. In this static TEP problem, there are two equality constraints, which is node balance of AC active and reactive power flow. In contrast, there are several inequality constraints to be considered, namely power flow limit on transmission lines constraint, active and reactive power generation limit, injection right of way constraint and bus voltage limit.

The TEP based on AC-OPF problem can be mathematically expressed as follows:

Minimize $F(x,u)$

Subject to $b(x,u) = 0; \ w_{\min} \leq w \leq w_{\max}$

Where: $F$ is an objective function which has to be minimized

$x$ is a static vector which denotes the dependent variables

$v$ is a vector representing all control variables

$b$ is equality constraint which is active and reactive power equilibrium condition

$w$ is inequality constraints which represents limits of control variables and system operating limits.

A. Objective Functions

Minimization of investment cost: The objective of this function is to minimize the investment cost IC of the system and this objective can be formulated as follows:

$$IC = \sum_{g=1}^{\text{ng}} c_g n_g$$

Where IC is the investment cost, $c_g$ is the cost of the candidate circuit for addition to the branch $i$-$j$ and $n_g$ is number of circuits added to the branch $i$-$j$ and $nl$ is the number of candidate circuits.

Minimization of operation cost: The objective of this function is to minimize the operation cost OC of the system and this objective can be formulated as follows:

$$OC = \sum_{g=1}^{\text{ng}} (a_g P_g^2 + b_g P_g + c_g)$$

Where OC is the operation cost (fuel cost), $P_g$ is the power generation of $i^{th}$ generator, $a_g$, $b_g$ and $c_g$ are the constant coefficients of power generation

Minimization of real power loss: The objective of this function is to minimize the real power loss PL of the system and this objective can be formulated as follows:

$$PL = \sum_{j=1}^{n} G_j (n_j) (v_i^2 + v_j^2 - 2 v_i v_j \cos(\delta_i - \delta_j))$$

Where $PL$ is the real power loss of the system, $n_j$ is the number of candidate lines between buses $i$ and $j$, $v_i$, $v_j$ are the bus voltages and $\delta_i$, $\delta_j$ are the voltage angles at buses $I$ and $j$ respectively.

State Variables: The state variables of a TEP based on AC-OPF consists of active power generation at slack bus $(P_{g_{sl}})$, voltage magnitude and phase angles of all load buses $(v_i, \delta_i)$, generator reactive power outputs $(Q_{g_{i}})$, transmission line loadings $(S_{ij})$. The state vector $X$ can be expressed as:

$$X = [P_{g_{1}}, V_{1}, ..., V_{n}, Q_{g_{1}}, ..., Q_{g_{ng}}, S_{11}, ..., S_{nn}]$$

where $nl$, $n$ and $ng$ represents the number of transmission lines, number of buses and number of generators respectively.

Control Variables: The control variables of TEP based on AC-OPF consists of active power generation output $(P_{g_{i}})$, number of additional lines $(n_g)$ and reactive power injections $(q_{ci})$. The control vector $u$ can be represented as:

$$u = [P_{g_{2}}, ..., P_{g_{ng}}, n_{l1}, ..., n_{lg}, q_{c1}, ..., q_{cnq}]$$

where $ng$, $nl$ and $nq$ are the number of generators, number of transmission lines and number of reactive power injections.

B. Constraints

The TEP based on AC-OPF has to follow equality and inequality constraints strictly. The real and reactive power equilibrium is considered as equality constraints and various operating limits are considered as inequality constraints.

Equality constraints: These constraints are shows load flow equations of both active power and reactive power. Mathematically represented as follows:

$$\sum_{j=1}^{n} V_j (G_j (n) \cos \delta_j + B_j (n) \sin \delta_j) - P_g + P_2 = 0$$

$$\sum_{j=1}^{n} V_j (G_j (n) \sin \delta_j + B_j (n) \cos \delta_j) - Q_g - Q_{2} = 0$$

where $P_g$, $Q_g$ are active and reactive power generations at $i^{th}$ bus respectively. $P_{a_{i}}$, $Q_{a_{i}}$ are active and reactive power demands at $i^{th}$ bus respectively. $q_{ci}$ is injected reactive power at $i^{th}$ bus.
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\( \mathbf{n} \) is number of added lines vector. \( \mathbf{G} \) and \( \mathbf{B} \) are the conductance and susceptance matrices respectively and are given by

\[
\mathbf{G}(n) = \begin{cases} 
G_{ij}(n) = -(n_i g_{ij} + n_j g_{ji}), \\
G_{ii}(n) = \sum_{j \neq i} (n_i g_{ij} + n_j g_{ji}) 
\end{cases}
\]

\( \mathbf{B}(n) = \begin{cases} 
B_{ij}(n) = -(n_i b_{ij} + n_j b_{ji}), \\
B_{ii}(n) = \sum_{j \neq i} (n_i b_{ij} + n_j b_{ji}) 
\end{cases}
\]

where \( g_{ij} \) and \( b_{ij} \) are conductance and susceptance of the transmission line connected between buses \( i \) and \( j \) at initial conditions and \( g_{ij} \) and \( b_{ij} \) are conductance and susceptance of the transmission line connected between buses \( i \) and \( j \) at added line conditions. \( n_i \) and \( n_j \) are the number of lines between buses \( i \) and \( j \) at initial and added line condition respectively.

**Inequality constraints:** These constraints are representation the system operating limits and are as follows:

**Generation constraints:** Generator voltages, real power outputs and reactive power outputs should be within their minimum and maximum limits.

\[
v_{g_{min}} \leq v_{g} \leq v_{g_{max}}, \quad \forall g = 1, \ldots, ng
\]

\[
P_{g_{min}} \leq P_{g} \leq P_{g_{max}}, \quad \forall g = 1, \ldots, ng
\]

\[
Q_{g_{min}} \leq Q_{g} \leq Q_{g_{max}}, \quad \forall g = 1, \ldots, ng
\]

**Reactive power injection limits:** Reactive power injection at load buses should be within minimum and maximum limits

\[
q_{a_{min}} \leq q_{a} \leq q_{a_{max}}, \quad \forall a = 1, \ldots, nq
\]

**Security limits:** These constraints include the limits of voltage magnitudes at all load buses, voltage angles at all buses and transmission line loadings of candidate lines.

\[
v_i \leq v_i \leq v_{i_{max}}, \quad i = 1, \ldots, npq
\]

\[
\delta_{i_{min}} \leq \delta_i \leq \delta_{i_{max}}, \quad i = 1, \ldots, nb
\]

\[
(n+n^a)S_{ij_{min}} \leq (n+n^a)S_{ij_{max}}, \quad l = 1, \ldots, n^l
\]

**Number of lines limits:** The number of lines added in the candidate lines should not exceed the maximum number of lines

\[
0 \leq n_i \leq n_{i_{max}}, \quad i = 1, \ldots, n^l
\]

**IV. DE ALGORITHM FOR TEP PROBLEM**

The proposed optimization program is expected to be able to solve a number of mathematical and engineering problems, such as economic power dispatch, unit commitment, optimal power flow, power system planning, transmission expansion planning, etc. The overall procedure of the DEA optimization program for TEP based on AC-OPF has been described as follows:

**Step 1:** Set up all required parameters of the DEA optimization process i.e. Set up control parameters of the DEA optimization process that are population size \( NP \), scaling mutation factor \( F \), crossover probability \( CR \), convergence criterion \( \varepsilon \), number of problem variables \( D \), lower and upper bounds of initial population \( x_{i_{min}}^m \) and \( x_{i_{max}}^m \) and maximum number of iterations or generations \( G_{max} \);

**Step 2:** Set generation \( G = 0 \) for initialization step of DEA optimization process;

**Step 3:** Initialization step i.e. Initialize population \( P \) of individuals according to equation (24) where each decision parameter in every vector of the initial population is assigned a randomly selected value from within its corresponding feasible bounds;

\[
x_i = x_{i_{min}}^m + \text{rand} \cdot (x_{i_{max}}^m - x_{i_{min}}^m)
\]

**Step 4:** For each generation check the equality criteria i.e. sum of all generations should be equal to load demand

\[
\sum_{i=1}^{n} P_i = P_d
\]

**Step 5:** Run optimum power flow

**Step 6:** Calculate and evaluate the fitness values of the initial individuals according to the problem’s fitness function;

**Step 7:** Rank the initial individuals according to their fitness;

**Step 8:** Set iteration \( G = 1 \) for optimization step of DEA optimization process;

**Step 9:** Apply mutation, crossover and selection operators to generate new individuals using equations (26 – 28)

\[
v_i^{(G)} = x_i^{(G)} + F \cdot (x_{ij2}^{(G)} - x_{ij3}^{(G)})
\]

\[
U_{ij}^{(G)} = U_{ij}^{(G)} \begin{cases} 
U_{ij}^{(G)} \quad \text{if } \text{rand}_j(0,1) \leq CR \quad \text{or} \quad j = s \\
x_{ij}^{(G)} \quad \text{otherwise}
\end{cases}
\]

\[
x_{ij}^{(G+1)} = \begin{cases} 
x_{ij}^{(G)} \quad \text{if } f(U_{ij}^{(G)}) \leq f(x_{ij}^{(G)}) \\
x_{ij}^{(G)} \quad \text{otherwise}
\end{cases}
\]

**Step 10:** For each generation check the equality criteria i.e. sum of all generations should be equal to load demand
\[ \sum_{i=1}^{n} P_i = P_g \]  
(29)

Step 11: Run optimum power flow

Step 12: Calculate and evaluate the fitness values of new individuals according to the problem’s fitness function;

Step 13: Rank new individuals by their fitness;

Step 14: Update the best fitness value of the current iteration (gbest) and the best fitness value of the previous iteration (pbest)

Step 15: Check the termination criteria; i.e. If \[ |X_{gbest} - X_i| > \varepsilon \] or \[ |pbest - gbest| > \varepsilon \] but the number of current generation remains not over the maximum number of generations \( G < G_{max} \), set \( G = G + 1 \) and return to step 9 for repeating to search the solution. Otherwise, stop to calculate and go to step 16;

Step 16: Output gbest of the last iteration as the best solution of the problem.

V. SIMULATION RESULTS

The Transmission Expansion Planning is executed using Differential Evaluation Algorithm has been implemented in MATLAB 7.10.0.499, 32 bit, 3 GB RAM, Intel Core 2 Duo T6600 2.20 GHz Processor with Windows 7 Operating System. Garver 6 – bus standard electrical transmission network is considered for the expansion planning. The TEP problem has been investigated in three case studies. The objective functions are investment cost, operation cost and power losses are tested in case studies. The generated power at each generator varies between \( P_{g_{min}} \) and \( P_{g_{max}} \).

Garver 6 – bus system consists of 6 buses, 15 possible branches, 760 MW, 152 MVAR demand and maximum 5 lines can be added to each branch. The complete data of the system is available in [3] and also shown in Fig.1. The dotted lines represent new possible line additions and solid lines are the existing lines. The detailed data of the test system was available in the above paper. Reactive power injections are at all the load buses 2, 4 and 5, the upper and lower limit of bus voltages are 1.05 and 0.95 p.u respectively are considered. The minimum generation \( g_{i_{min}} \) of each generator is considered as zeros, the maximum generation \( g_{i_{max}} \) of each generator, resistance and reactance of candidate line, permissible line loading of each line is given in [14]. To obtain optimum values of all objective functions, the algorithm was run for 10 times. DE parameter are \( \text{D}=15, \text{NP}=50, \text{F}=0.358, \text{CR}=0.458 \) and maximum iterations are 500.

TEP has been studied for Garver – 6 bus system by considering three (3) case studies as are follows:

Case – I: Reactive power is not injected at any load bus, investment cost $130 million obtained in base case [12].

Case – II: The selection of reactive power injections at all load buses has been varied between minimum and maximum limits (0 MVAR and 0.1 MVAR respectively). Thus the investment cost $110 million obtained in base case [13].

Case – III: The selection of reactive power injections at all load buses has been varied between minimum and maximum limits (0 MVAR and 1.5 MVAR respectively) so that investment cost has been decreased to $ 80 million compared to Case – I and Case – II.

![Fig.1 IEEE 6-bus Garver test system](image)

A. Case- I results

Transmission Expansion Planning based on AC-OPF using AC network. TEP using DEA with various DE parameters and system parameter and constraints are considered. In this case reactive power is not injected at any load buses. The results are tabulated are shown in TABLE 1 for investment cost, operation cost and real power loss as the objective function simultaneously.

Without injection of reactive power at load buses, investment cost as objective function, investment cost $ 130 million was in base case and the optimum investment cost $ 80 million was resulted in combination of flat frequency control generator model and constant impedance load model i.e. FCC+CL.

By maintaining the system security and all constraint limits, operation cost as the objective function, optimum operation cost $593.4596 /h resulted in combination of flat frequency control generator model and constant impedance load model i.e. FCC+CL.
model and constant impedance load model (FFC+CZ LM). The healthy system voltage profile has been maintained and also line loadings are within the permissible limits by without injection of reactive power at load buses.

Power loss as the objective function, the optimum loss 2.8507 MW resulted in combination of flat frequency control generator model and constant impedance load model (FFC+CZ LM).

<table>
<thead>
<tr>
<th>TABLE 1: SUMMARIZED RESULTS OF GRAVER 6-BUS SYSTEM FOR CASE - I</th>
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</thead>
<tbody>
<tr>
<td>Model/Objective Function</td>
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<tr>
<td>--------------------------</td>
</tr>
<tr>
<td>Base Case</td>
</tr>
<tr>
<td>CP LM</td>
</tr>
<tr>
<td>CC LM</td>
</tr>
<tr>
<td>CZ LM</td>
</tr>
<tr>
<td>Mixed LM</td>
</tr>
<tr>
<td>FFC</td>
</tr>
<tr>
<td>FFC + CP LM</td>
</tr>
<tr>
<td>FFC + CZ LM</td>
</tr>
<tr>
<td>FFC + Mixed LM</td>
</tr>
</tbody>
</table>

B. Case- II results

The reactive power injection at all load buses was varied between upper and lower limits (0 MVAR and 0.1 MVAR respectively). The optimum values of investment cost, operation cost and power loss as objective function are presented in TABLE 2.

With injection of reactive power at all load buses, investment cost $ 110 million was in base case where as the optimum investment cost $ 80 million resulted in (a) constant impedance load model (CZ LM), (b) combination of flat frequency control generator model and constant current load model (FFC+CC LM) and (c) combination of flat frequency control generator model and constant impedance load model (FFC+CZ LM).

By maintaining the system security and all constraint limits, operation cost as the objective function, optimum operation cost 593.4513 $/h resulted in combination of flat frequency control generator model and constant impedance load model (FFC+CZ LM). The healthy system voltage profile has been maintained at all buses and also line loadings are within the permissible limits in all lines.

Power loss as the objective function, the optimum real power loss 2.8188 MW resulted in combination of flat frequency control generator model and constant impedance load model (FFC+CZ LM).

C. Case- III results

In this case study to obtain better results, compared to case – I and case – II the reactive power injections are varied between upper and lower limits (0 MVAR and 1.5 MVAR respectively) at all load buses. Summarized results of investment cost, operation cost and power loss are objective functions are shown in TABLE 3.

<table>
<thead>
<tr>
<th>TABLE 2: SUMMARIZED RESULTS OF GRAVER 6-BUS SYSTEM FOR CASE - II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model/Objective Function</td>
</tr>
<tr>
<td>--------------------------</td>
</tr>
<tr>
<td>Base Case</td>
</tr>
<tr>
<td>CP LM</td>
</tr>
<tr>
<td>CC LM</td>
</tr>
<tr>
<td>CZ LM</td>
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<tr>
<td>Mixed LM</td>
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<tr>
<td>FFC</td>
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<tr>
<td>FFC + CP LM</td>
</tr>
<tr>
<td>FFC + CC LM</td>
</tr>
<tr>
<td>FFC + CZ LM</td>
</tr>
<tr>
<td>FFC + Mixed LM</td>
</tr>
</tbody>
</table>

Reactive power injected at all load buses are varied between minimum and maximum limits such that bus voltages are within limits and line loadings are also within their limits. Investment cost $ 80 million was in base case and also the optimum investment cost $ 80 million resulted in (a) base case, (b) constant power load model (CP LM), (c) constant current load model (CC LM), (d) mixed load model (mixed LM), (e) flat frequency control generator model (FFC), (f) combination of flat frequency control generator model and constant power load model (FFC+CP LM), (g) combination of flat frequency control generator model and constant current load model (FFC+CC LM), (h) combination of flat frequency control generator model and constant impedance load model (FFC+CZ LM), (i) combination of flat frequency control generator model and mixed load model (FFC+mixed LM).

In addition of reactive power injection at all load buses, the power generations are varied by each generator within their minimum and maximum limits.
to maintain system healthy conditions. Operation cost as the objective function, optimum operation cost 593.4513$/h resulted in combination of flat frequency control generator model and constant impedance load model (FFC+CZ LM).

Likely the system was stabilized with respect to all equality and inequality conditions with different constraints. Power loss as the objective function, the optimum power loss 2.7851 MW resulted in combination of flat frequency control generator model and constant impedance load model (FFC+CZ LM).

### TABLE 3: SUMMARIZED RESULTS OF GRAVER 6-BUS SYSTEM FOR CASE - III

<table>
<thead>
<tr>
<th>Model/Objective Function</th>
<th>Investment Cost ($)</th>
<th>Operation Cost ($)</th>
<th>Power Loss (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>80</td>
<td>614.526</td>
<td>2.8266</td>
</tr>
<tr>
<td>CP LM</td>
<td>80</td>
<td>614.526</td>
<td>2.8262</td>
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<tr>
<td>CC LM</td>
<td>90</td>
<td>608.6647</td>
<td>2.8138</td>
</tr>
<tr>
<td>CZ LM</td>
<td>110</td>
<td>594.395</td>
<td>2.7998</td>
</tr>
<tr>
<td>Mixed LM</td>
<td>80</td>
<td>612.5743</td>
<td>2.8205</td>
</tr>
<tr>
<td>FFC</td>
<td>80</td>
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<td>FFC + CP LM</td>
<td>80</td>
<td>614.5754</td>
<td>2.8315</td>
</tr>
<tr>
<td>FFC + CC LM</td>
<td>80</td>
<td>608.8142</td>
<td>2.8119</td>
</tr>
<tr>
<td>FFC + CZ LM</td>
<td>80</td>
<td>593.4513</td>
<td>2.7851</td>
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<tr>
<td>FFC + Mixed LM</td>
<td>80</td>
<td>612.2526</td>
<td>2.8236</td>
</tr>
</tbody>
</table>

**CONCLUSIONS**

This paper explored the possibility of applying AC-OPF based models to the TEP problem. Nine TEP models are presented in this paper. The formulation of each model is shown and discussed in detail. A validation process guarantees the resultant TEP plan is strictly AC feasible. The conclusions of this paper are: The AC model can be applied to model TEP. The solution of DEA-based AC-TEP models is still challenging. By reformulation and relaxation, it is possible to solve the DEA-based ACTEP problem and obtain an optimal solution.

**REFERENCES**


