SIMILARITY SOLUTION TO STUDY THE EFFECT OF VARIABLE VISCOSITY ON NON-NEWTONIAN BUOYANCY INDUCED FLOW OVER AN AXISYMMETRIC BODY IMMERSED IN A POROUS MEDIUM SATURATED BY A NANOFLUID

SHOBHA BAGAI

Cluster Innovation Centre, University of Delhi, New Delhi, India - 110007

Abstract— The study presents the effect of temperature dependent viscosity on natural convection heat transfer of nanofluids over an axisymmetric body embedded in a saturated non-Darcy porous medium. Similarity variables are introduced to reduce the governing partial differential equations to a system of highly coupled nonlinear ordinary differential equations. The model incorporates the non-dimensional parameters: buoyancy ratio Nr, Brownian motion parameter Nb, thermophoresis parameter Nt and Lewis number Le. The local wall heat flux distribution is discussed for three geometries, namely vertical flat plate, horizontal cylinder and sphere for pseudo plastic fluids, Newtonian fluid and dilatant fluids

Keywords— Nanofluid, Non-Newtonian, Porous medium, Variable viscosity

I. INTRODUCTION

Nanofluid refers to a liquid containing suspended nanoparticles. The coining of the term ‘nanofluid’ for such fluids is credited to Choi [1]. Nanofluids property of enhanced thermal conductivity is being currently explored to enhance heat transfer. Masuda, Ebata, Teramae and Hishinuma [2] reported the earliest investigation of thermal conductivity enhancement in liquid dispersions of nanoparticles. Buongiorno [3] gave a detailed explanation of the abnormal convective heat transfer enhancement observed in nanaofluids. The study of convective heat transfer over bodies embedded in porous media has gained attention in the last few years. Nield and Kuznetsov [4] gave an analytical study of the onset of convection in a horizontal layer of a porous medium saturated by a nanofluid. Their model incorporated the effect of Brownian motion and thermophoresis. They also studied the Cheng-Minkowycz problem [5] of natural convection past a vertical plate in a porous medium saturated with nanofluid. Darcy model was employed for the porous medium. Hady, Ibrahim, Abdel-Gaied and Eid [6] gave similarity solutions for the non-Newtonian flow over an isothermal vertical plate in porous medium saturated with nanofluid. The problem of natural convective boundary layer flow over a horizontal plate embedded in porous medium saturated with a nanofluid was investigated by Gorla and Chamkha [7]. More recently they also studied the problem of natural convective boundary layer flow over a non-isothermal vertical plate embedded in a porous medium saturated with a nanofluid [8]. The problem with prescribed heat flux was investigated by Noghrehabadi, Behseresht and Ghalambaz [9]. The problem of steady, laminar, mixed convection flow of a non-Newtonian fluid past a vertical flat plate embedded in a porous medium saturated with a nanofluid was considered by Rashad, Chamkha and Abdou [10]. Bhaduria, Agarwal and Kumar[11] investigated the problem of linear and non-linear thermal instability in a horizontal porous medium saturated by nanofluid. The Darcy-Forchheimer model was used by Udin and Harmand[12] to investigate the unsteady natural convection heat transfer of nanofluid along a vertical plate embedded in a porous medium. Apart from the vertical and horizontal geometry, Hady, Ibrahim, Abdel-Gaied and Eid presented the problem of boundary layer flow in a porous medium of a nanofluid past a vertical cone [13]. In all the work cited above it is assumed that the viscosity of the fluid is constant. But, the viscosity of most of the fluids generally decreases with an increase in the temperature. To incorporate this effect it is assumed the viscosity decreases exponentially with the temperature. Moreover, this study also aims to explore the effect of viscosity on non-Newtonian buoyancy induced flow past an arbitrary shaped axisymmetric body immersed in a nanofluid saturated porous medium. The effect of viscosity on heat flux for vertical flat plate, horizontal cylinder and sphere is discussed.

Nomenclature

c	=	specific heat at constant presure
D_B	=	Brownian diffusion coefficient
D_T	=	thermophoretic diffusion coefficient
f	=	dimensionless stream function
g	=	acceleration due to gravity
K^*	=	modified permeability
k_m	=	thermal conductivity of porous medium
Le	=	Lewis number
n	=	Power law constant
q'''	=	internal heat generation per unit volume
q	=	heat flux
r^*	=	function representing wall geometry
Ra_x	=	local Rayleigh number
T	=	temperature
u, v	=	velocity components in x- and y- direction
x, y	=	boundary layer cordinates
Greek Symbols		
β	=	coefficient of thermal expansion
ϕ	=	nanoparticle volume fraction
η	=	similarity variable
ν^*	=	kinematic viscosity
θ	=	dimensionless temperature
ρ	=	density
ω	=	rescaled nanoparticle volume fraction
ψ	=	Stream function
γ	=	viscosity parameter
Subscripts		
f	=	physical property related to fluid
p	=	physical property related to porous medium
w	=	wall condition
∞	=	ambient condition

II. GOVERNING EQUATIONS AND MATHEMATICAL ANALYSIS

Consider a two dimensional problem of an axisymmetric body immersed in a non-Newtonian nanofluid saturated porous medium. The x- coordinate is measured along the body and the y- coordinate is along the normal. The body is maintained at a wall temperature T_w (x). It exceeds the ambient temperature T_∞ everywhere. The nanoparticle fraction at the surface of the body is assumed to be constant and it takes the value ϕ_w. Assuming the homogeneity and local thermal equilibrium of the porous medium, the governing equations under Oberbeck-Boussinesq approximation can be written as

$$\frac{\partial}{\partial x}\left(r^* u\right)+\frac{\partial}{\partial y}\left(r^* v\right)=0$$

$$\frac{\partial}{\partial y}\left(u^n\right)=\frac{(1-\phi_\infty)\beta g_x K^*}{\nu^*}\frac{\partial T}{\partial y}-\frac{\left(\frac{\rho_p}{\rho_{f_\infty}}-1\right)g_x K^*}{\nu^*}\frac{\partial \phi}{\partial y}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial \phi}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]$$

$$u\frac{\partial \phi}{\partial x}+v\frac{\partial \phi}{\partial y}=\varepsilon\left[D_B\frac{\partial^2 \phi}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}\right] \qquad (4)$$

where $\alpha_m=\frac{k_m}{(\rho c)_f}$ is the thermal diffusivity of the porous medium and $\tau/\varepsilon=\frac{(\rho c)_p}{(\rho c)_f}$ is the ratio of the effective heat capacity of the porous medium to the effective heat capacity of the nanoparticle material. The function $r^*(x)$ describes the geometric configuration of the body and is defined by

$$r^*(x)=\begin{cases}1 & :\text{plane flow}\\ r(x) & :\text{axisymmetric flow}\end{cases}$$

The acceleration due to gravity g_x along the x- component is related to the body shape function $r^*(x)$ by

$$g_x=\left[1-\left(\frac{dr}{dx}\right)^2\right]^{1/2}$$

Equations (1) – (4) embody the conservation of mass, momentum, thermal energy and nanoparticles respectively. The above equations coupled with the boundary conditions

$$v=0,\quad T=T_w(x),\quad \phi=\phi_w \quad \text{at } y=0 \qquad (5)$$

$$u\to 0,\quad T\to T_\infty,\quad \phi\to\phi_\infty \quad \text{as } y\to\infty \qquad (6)$$

are solved for the field variables u and v, the velocity components; T, the temperature and ϕ, the nanoparticle volume fraction.

Integrating equation (2) once and using the boundary conditions (5) and (6) yields

$$u^n=\frac{(1-\phi_\infty)\beta g_x K^*}{\nu^*}(T-T_\infty)-\frac{\left(\frac{\rho_p}{\rho_{f_\infty}}-1\right)g_x K^*}{\nu^*}(\phi-\phi_\infty) \qquad (7)$$

It is assumed that the viscosity exponentially decays with the temperature

$$\nu^*=\nu_0\ e^{-b(T-T_\infty)}$$

Equation (1) is satisfied by introducing the stream function ψ defined by

$$u=\frac{1}{r^*}\frac{\partial\psi}{\partial y},\qquad v=-\frac{1}{r^*}\frac{\partial\psi}{\partial x} \qquad (8)$$

On introducing the general transformations, where $\Delta T_w=T_w-T_\infty$ and $\Delta\phi_w=\phi_w-\phi_\infty$,

$$\psi=\alpha_m r^*\left(Ra_x I(x)\right)^{1/2} f(\eta) \qquad (9)$$

(1) $$T-T_\infty=\Delta T_w\theta(\eta) \qquad (10)$$

$$\phi-\phi_\infty=\Delta\phi_w\omega(\eta) \qquad (11)$$

(2) $$\eta=\frac{y}{x}\left(\frac{Ra_x}{I(x)}\right)^{1/2} \qquad (12)$$

equations (7), (3) and (4) reduce to

(3) $$\left(f'(\eta)\right)^n=\left(\theta(\eta)-Nr\,\omega(\eta)\right)\ \exp\left(\gamma\theta(\eta)\right) \qquad (13)$$

$$\theta''(\eta)+\frac{1}{2}f(\eta)\theta'(\eta)+Nb\omega'(\eta)\theta'(\eta)+Nt(\theta'(\eta))^2=0 \qquad (14)$$

$$\omega''(\eta)+\frac{Nt}{Nb}\theta''(\eta)+\frac{1}{2}Lef(\eta)\omega'(\eta)=0 \qquad (15)$$

where Nr, Nb and Nt denote a buoyancy ratio, a Brownian motion parameter and a thermophoresis parameter. These parameters are defined as

$$Nr=\frac{\rho_p-\rho_{f_\infty}}{(1-\phi_\infty)\rho_{f_\infty}\beta}\frac{\Delta\phi_w}{\Delta T_w},$$

$$Nb=\frac{\tau D_B\Delta\phi_w}{\alpha_m},$$

$$Nt=\frac{\tau D_T\Delta T_w}{T_\infty\alpha_m}$$

In equations (15) $Le=\frac{\alpha_m}{\varepsilon D_B}$ is the Lewis number.

The dimensionless viscosity parameter γ is defined as

$$\gamma=b\Delta T_w$$

The integral function I(x) is given as

$$I(x)=\frac{\int_0^x r^{*2}g_x^{1/n}dx}{r^{*2}g_x^{1/n}x} \qquad (16)$$

and

$$Ra_x=\frac{x}{\alpha_m}\left(\frac{(1-\phi)_\infty g_x\beta K^*\Delta T_w}{\nu_0}\right)^{1/n}$$

is the Rayleigh number.

The system of non-linear differential equations (13) – (15)

Is coupled with the boundary conditions

$$\begin{aligned}&\text{At}\quad \eta=0; \quad f=0,\ \theta=1,\ \omega=1\\ &\text{As}\quad \eta\to\infty; \quad f'\to 0,\ \theta\to 0,\ \omega\to 0\end{aligned} \qquad (17)$$

III. RESULTS AND DISCUSSIONS

Although the system (13) – (17) is valid for any arbitrary geometry of the embedded body, three geometries are considered in the present study, namely, vertical flat plate, horizontal cylinder and a sphere. For these three geometries, the integral in equation (16) can be written as

$$I(x)=\begin{cases}1, & \text{Vertical Plate [VP]}\\ \dfrac{\int_0^\phi \sin^{1/n}\phi\ d\phi}{\phi\sin^{1/n}\phi}, & \text{Horizontal Cylinder [HC]}\\ \dfrac{\int_0^\phi \sin^{2+1/n}\phi\ d\phi}{\phi\sin^{2+1/n}\phi}, & \text{Sphere [S]}\end{cases} \qquad (18)$$

where $\phi=\sin^{-1}\left(\frac{x}{r}\right)$ and r is the radius of the cylinder or the sphere.

The non-dimensional local heat flux defined by

$$q^*=\left(\frac{q_wL_r}{\Delta T_wk_m}\right)\cdot\left(\frac{(1-\phi_\infty)K^*g\beta\Delta T_wL_r^{1/n}}{\alpha^n\nu_0}\right)^{-1/2n} \qquad (19)$$

for the three geometries is given by

$$q^*=\begin{cases}-\theta'(0)\ x^{-1/2}\ L_r^{1/2} & \text{[VP]}\\ -\theta'(0)\ \sin^{1/n}\phi\left[\int_0^\phi \sin^{1/n}\phi\ d\phi\right]^{-1/2} & \text{[HC]}\\ -\theta'(0)\ \sin^{1+1/n}\phi\left[\int_0^\phi \sin^{2+1/n}\phi\ d\phi\right]^{-1/2} & \text{[S]}\end{cases} \qquad (20)$$

In equation (19), q_w is the local surface heat flux defined by $q_w=-k_m\frac{\partial T}{\partial y}\Big|_{y=0}$ and L_r is the reference length. The heat flux for a flat plate at x = L_r are given in Table 1 for a typical case of Nt = Nb = Nr = 0.5 and Le = 10. It is observed that the heat flux decreases as the viscosity parameter γ increases for a given value of n. Although for smaller value of the viscosity parameter γ the heat flux decreases with an increase in the value of n, a reverse process is observed for higher values of n. That is, the heat flux increases as the value of n increases. A similar result is observed in [14]. The heat flux over a cylinder and sphere for Le = 10, Nt = Nb = Nr =0.5, are depicted in Fig. 1 and Fig. 2 respectively. For γ = 0, the heat flux decreases as the value of n increases. But as we increases the value of γ , there exists some critical $\phi=\phi_1$ and $\phi=\phi_2$ between the front and rear stagnation point such that the heat flux increases as n increases for $\phi_1\le\phi\le\phi_2$. For $\phi<\phi_1$ and $\phi>\phi_2$ the heat flux decreases as n increases. Plots of the velocity profile f ′(η), temperature profile θ(η) and nanoparticle volume fraction ω(η) for n = 0.5, 1.0, 2.0 are shown in Fig. 3 – Fig. 5 for Le = 10, Nt = Nb = Nr = 0.5. For constant viscosity (γ = 0) the velocity is less for all values of η for smaller values of n. But as the value of the parameter γ increases we observe that the value of f ′(η) increases for smaller value n and then decreases as n decreases. The temperature profile decreases as n increases for γ = 0. As the value of γ increases, the temperature profile increases with an increase in n. Similar behaviour is observed for the nanoparticle volume fraction ω(η).

Table 1: Variation of $[-\theta'(0)]$ with γ and n for $Nt=Nb=Nr=0.5$, $Le=10$

	$\gamma=0$	$\gamma=0.1$	$\gamma=0.5$	$\gamma=0.75$	$\gamma=1$
$n=0.5$	-0.1663	-0.1797	-0.2473	-0.3033	-0.3730
$n=1.0$	-0.2171	-0.2254	-0.2626	-0.2893	-0.3190
$n=2.0$	-0.2520	-0.2565	-0.2762	-0.2893	-0.3031

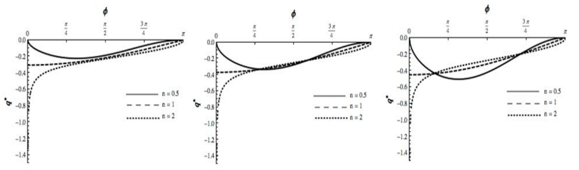

Fig. 1(a) Fig. 1(b) Fig. 1(c)

Fig.1: Local heat flux for a cylinder (a) $\gamma = 0$ (b) $\gamma = 0.5$ (c) $\gamma = 1$

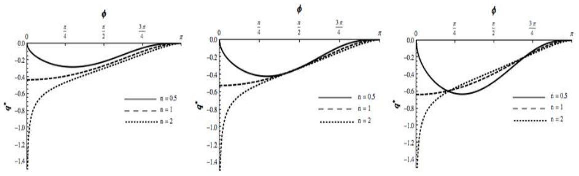

Fig. 2(a) Fig. 2(b) Fig. 2(c)

Fig.2: Local heat flux for a sphere (a) $\gamma = 0$ (b) $\gamma = 0.5$ (c) $\gamma = 1$

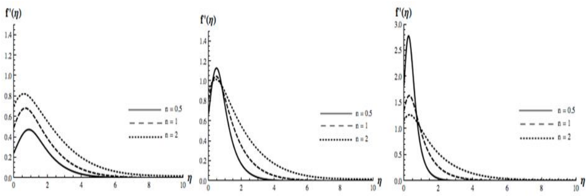

Fig. 3(a) Fig. 3(b) Fig. 3(c)

Fig.3: Plot of f '(η) for different values of n (a) $\gamma = 0$ (b) $\gamma = 0.5$ (c) $\gamma = 1$

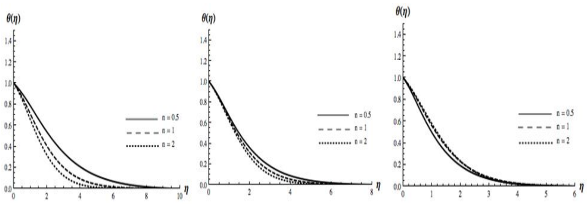

Fig. 4(a) Fig. 4(b) Fig. 4(c)

Fig.4: Plot of θ (η) for different values of n (a) $\gamma = 0$ (b) $\gamma = 0.5$ (c) $\gamma = 1$

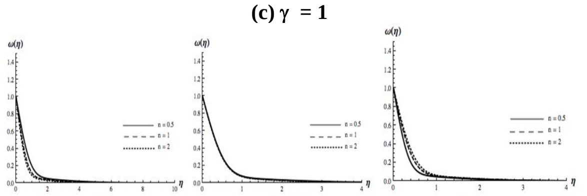

Fig. 5(a) Fig. 5(b) Fig. 5(c)

Fig.5: Plot of ω (η) for different values of n (a) $\gamma = 0$ (b) $\gamma = 0.5$ (c) $\gamma = 1$

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