GEOMETRICALLY NONLINEAR ANALYSIS OF COMPOSITE PLATE

GURANGOUDA S. PATIL, ANAND P. YALAWAR, SHRAVANKUMAR B. KERUR

1PG-Student, 2Associate professor, Dept. of Mechanical Engineering, Basaveshwar Engineering College, Bagalkot-587103

Abstract- Composites are being widely used in aeronautical, mechanical, civil and chemical industries due to their high strength to stiffness and low weight to stiffness ratios. This present paper discusses, the geometrically nonlinear behavior of composite plate with various boundary conditions. The finite element formulations is based on the first order shear deformation theory (FSDT). The finite element analysis has been carried out for a composite plate using an eight nodded quadrilateral element and five degree of freedom (DOF). Nonlinear equilibrium equations are linearised by using Newton-Raphson method. Numerical analysis has been carried out by developing program in analysis software ANSYS and various results are obtained. The result are compared with those available in the literature. The present paper emphasis on the effect of number of layers, stacking sequence and different boundary condition on geometrically nonlinear deflections composite plates.

Keywords- FSDT, composite, geometric nonlinearity, cross-ply

I. INTRODUCTION

Composite materials are preferred in places where lighter materials are desired or required without sacrificing strength. They have even become essential for many applications. Composite materials are being used in a variety of applications such as the structural parts of aircrafts, automobiles, chemical equipment, transformer tubes, boats, etc. Some transmission gears make use of plastic materials in many different places such as watches, instruments, washing machines, gear pumps, etc. Composite materials are efficient in applications that required high strength to weight and stiffness to weight ratios. Brebbeia and Connor [1] presented the finite element displacement formulation applicable to arbitrary plate and shell elements for geometrically nonlinear problems and also developed appropriate equation for Newton-Raphson iteration. Wood and zienkiewicz [2] studied the geometrically nonlinear analysis of beams frames and arches using total Lagrangian coordinate system by using modified isoparametric element and system non lateral equation was solved using Newton-Raphson method. Bathe and Bolourchi [3] presented the finite element method for linear, geometric, and material nonlinear analysis of plates and shell. Pica et al. [4] presented a geometrically nonlinear analysis of plates using finite element Mindilin formulation. Reddy et.al [5] developed a finite element model based on the combined theory of the yang, Norris, Stavsky and Von Karman [6] that, it accounts for the transverse shear strain, large rotations. Puctha and Reddy [7] developed a mixed shear flexible finite element with relaxed continuity, geometrically linear and nonlinear analysis of laminated anisotropic plates. Chia [8] solved the nonlinear bending of an unsymmetrically laminated angle ply rectangular plate analytically under lateral load satisfying the von-karman type strain. Striz et al. [9] investigated the behavior of thin, circular isotropic elastic plates with the immovable edges and undergoing large deflections ,they used Newton-Raphson technique to solve the nonlinear systems of equations. Z. G. Azizian and D. J. Dawe [10] studied geometrically nonlinear analysis of rectangular mindlin plates using the finite strip method. Singh et al. [11] investigated the large deflections bending analysis of anti-symmetric rectangular cross-ply plates based on von-karman plate theory, with one-term approximation for the in-plane & transverse displacements, under sinusoidal loading. Clarke et al. [12] described various incremental- iterative technique based on the Newton-Raphson approach to analyze the geometric nonlinear behavior.

II. FINITE ELEMENT FORMULATION AND METHODOLOGY

The nonlinear finite element modelling and analysis is more complicated than linear one, as the stiffness matrix in nonlinear equilibrium equations is dependent on deformations. The first order shear deformation plate theory is considered in the finite element formulation. An eight- noded isoparametric plate/shell element is proposed for the geometrically nonlinear analysis of composite plates subjected to mechanical loading. The geometry and origin of the material coordinates are at the middle of the laminate as shown in Fig.1.

Fig.1 Geometry of the laminated composite plate
III. DISPLACEMENT FIELD

The first order shear deformation theory is used to describe the kinematics of deformation for the present analysis. The displacements components \( u, v, w \) along \( x, y, z \) respectively are expressed as

\[
\begin{align*}
    u(x, y, z, t) &= u_0(x, y, t) + z\Theta_z(x, y, t) \\
    v(x, y, z, t) &= v_0(x, y, t) + z\Theta_x(x, y, t) \\
    w(x, y, z, t) &= w_0(x, y, t)
\end{align*}
\]

\( u, v, w \) are total mechanical displacement at any point in the laminate along \( x, y, z \) axes respectively; \( u_0, v_0, w_0 \) are mid-plane \((z=0)\) displacements in \( x, y, z \) axes respectively. \( \Theta_z \) and \( \Theta_y \) are rotations of normal to the mid plane about \( x \) and \( y \) axis respectively.

Strain-displacement relation

Shear deformable plate theory that include von man type geometrically nonlinearity are derived. Sander’s nonlinear strain-displacements relations associated with the displacements field are used. The strain-displacement relations are given by

\[
\begin{align*}
    \varepsilon_{xx} &= \varepsilon_{0xx} + z\kappa_{xx} \\
    \varepsilon_{yy} &= \varepsilon_{0yy} + z\kappa_{yy} \\
    \gamma_{xy} &= \gamma_{0xy} + z\kappa_{xy} \\
    \gamma_{xz} &= \gamma_{0xz} \\
    \gamma_{yz} &= \gamma_{0yz}
\end{align*}
\]

Where the strains of the middle surface are related to the displacements by

\[
\begin{align*}
    \varepsilon_{0xx} &= \frac{\partial u}{\partial x} + \frac{1}{2}(\partial w / \partial x)^2 \\
    \varepsilon_{0yy} &= \frac{\partial v}{\partial y} + \frac{1}{2}(\partial w / \partial y)^2 \\
    \gamma_{0xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + (\partial w / \partial x)(\partial w / \partial y) \\
    \gamma_{0xz} &= \Theta_z + \frac{\partial w}{\partial x} \\
    \gamma_{0yz} &= \Theta_y + \frac{\partial w}{\partial y}
\end{align*}
\]

and the curvature are related as

\[
\begin{align*}
    \kappa_{xx} &= \frac{\partial \Theta_x}{\partial x} \\
    \kappa_{yy} &= \frac{\partial \Theta_y}{\partial y} \\
    \kappa_{xy} &= \frac{\partial \Theta_y}{\partial x} + \frac{\partial \Theta_x}{\partial y}
\end{align*}
\]

The total strain \( \{\varepsilon\} \) is the sum of linear \( \{\varepsilon_L\} \) and nonlinear \( \{\varepsilon_{NL}\} \) strain components and written as

\[
\{\varepsilon\} = \{\varepsilon_L\} + \{\varepsilon_{NL}\}
\]

The linear and nonlinear strain components are

\[
\begin{align*}
    \{\varepsilon_L\} &= \begin{bmatrix}
        \varepsilon_{xx}^L \\
        \varepsilon_{yy}^L \\
        \gamma_{xy}^L
    \end{bmatrix} = \begin{bmatrix}
        \frac{\partial u}{\partial x} + z\Theta_z/\partial x \\
        \frac{\partial v}{\partial y} + z\Theta_y/\partial y \\
        \Theta_z + \frac{\partial w}{\partial x}
    \end{bmatrix} \\
    \{\varepsilon_{NL}\} &= \begin{bmatrix}
        \varepsilon_{xx}^{NL} \\
        \varepsilon_{yy}^{NL} \\
        \gamma_{xy}^{NL}
    \end{bmatrix} = \begin{bmatrix}
        1/2(\partial w / \partial x)^2 \\
        1/2(\partial w / \partial y)^2 \\
        (\partial w / \partial x)(\partial w / \partial y)
    \end{bmatrix}
\end{align*}
\]

The membrane strains \( \{\varepsilon_{xx}, \varepsilon_{yy}, \gamma_{xy}\} \) are linear through the laminate thickness, while the transverse shear strains \( \{\gamma_{xz}, \gamma_{yz}\} \) are constant through the thickness of the laminate. It follows that the transverse shear stress are also constant. Hence, shear correction factors is introduced to account for the non-uniform distribution of transverse shear strain across the thickness of the laminate.

Constitutive relations

The laminated composite plate is assumed to consist of number of thin elastic substrate laminates as shown in Fig.1 the material of each lamina is composed of an arbitrary orientation of fibers embedded in matrix material. A lamina is assumed as elastically orthotropic. The constitute equation of substrate lamina having fibre orientation \( (\Theta) \) with respect to material axes system \((1-2-3)\) as shown in Fig.2 [13]may be expressed as below

\[\text{Fig. 2 Lamina with material axes system.}\]
Stress-strain relationship for a substrate lamina is given by

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} & 0 & 0 & 0 \\
Q_{12} & Q_{22} & 0 & 0 & 0 \\
0 & 0 & Q_{66} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
\]

(8)

Where

\[
Q_{11} = \frac{E_{11}}{1-\nu_{21}\nu_{21}}, \quad Q_{12} = \nu_{12}E_{22}, \quad Q_{22} = \frac{E_{22}}{1-\nu_{21}\nu_{21}},
\]

\[
Q_{66} = G_{12}, \quad Q_{44} = G_{11}, \quad Q_{55} = G_{23}
\]

(9)

The subscripts 1, 2, 3 represent the principle material axes. \(E_{11}, E_{22}, G_{12}, G_{13}, G_{23}\) are elastic longitudinal, transverse and shear moduli of laminated shell respectively. \(k_i\) is the correction factor.

The constitutive relation is written in short as

\[
\{\sigma\} = [Q]\{\varepsilon\}
\]

(10)

Where \(\{\sigma\}, [Q], \{\varepsilon\}\) are stress, strain, constitutive matrix respectively.

The global axis (x-y-z coordinate) lamina stiffness matrix are expressed as follows

\[
\begin{bmatrix}
\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\
\bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\
\bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\
0 & 0 & 0 & k_{1s}\bar{Q}_{44} & \bar{Q}_{45} \\
0 & 0 & 0 & \bar{Q}_{54} & k_{1s}\bar{Q}_{55}
\end{bmatrix}
\]

(11)

Where

\[
\bar{Q}_{11} = (Q_{11}m^4 + Q_{66}m^2 + 2Q_{44}m^2 + 2Q_{66}m^2 + 2Q_{66}m^2 + 2Q_{66}m^2 + 2Q_{66}m^2 + 2Q_{66}m^2)
\]

\[
\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{44})m^2 + Q_{12}(n^4 + m^4)
\]

\[
\bar{Q}_{22} = (Q_{11} + Q_{22} - 4Q_{44})m^2 + Q_{12}(n^4 + m^4)
\]

\[
\bar{Q}_{16} = (Q_{11} + Q_{12} - 2Q_{44})m^2 + (Q_{12} - Q_{22} + 2Q_{66})m^2
\]

\[
\bar{Q}_{26} = (Q_{11} + Q_{12} - 2Q_{44})m^2 + (Q_{12} - Q_{22} + 2Q_{66})m^2
\]

\[
\bar{Q}_{44} = Q_{44}m^4 + Q_{44}m^2
\]

\[
\bar{Q}_{45} = (Q_{55} - Q_{44})mn
\]

\[
\bar{Q}_{55} = Q_{55}m^2 + Q_{55}m^2
\]

(12)

Energy equation

The present analysis involves structural displacements due to external mechanical loading. The Potential energy of the system is given by

\[
T_p = \frac{1}{2} \int_{V} \{\varepsilon^k\}^T \{\sigma^k\} dv
\]

(19)

Substitute the equation (10) into (19), the above equation is rewritten as

\[
T_p = \frac{1}{2} \int_{V} \{\varepsilon^k\}^T [Q]\{\varepsilon^k\} dv
\]

(20)

IV. VARIATIONAL PRINCIPLE

The equilibrium configuration of the system can be determined from the following variational principle: an admissible force state satisfies the geometric constraints if, and only if, the variational indicator \(\delta T_p\) vanishes for arbitrarily admissible force variations. This principle is known as Principle of Minimum Potential Energy and is given as

\[
\delta T_p = \frac{1}{2} \int_{V} \{\delta \varepsilon^k\}^T [Q]\{\varepsilon^k\} dv
\]

(21)
V. FINITE ELEMENT FORMULATION

Geometrically nonlinear finite element is developed for the static analysis of laminated composite plate using first order shear deformation theory. An eight noded \( C^0 \) continuous element is employed.

Isoparametric element

In the present analysis, an eight noded quadrilateral isoparametric element is used. The independent field variables (basic unknowns) based on the displacements field are \( u, v, w, \theta_x, \theta_y \). The plate is discretized in to finite elements. The element configuration is shown in Fig. 3. The shape function of the element are given by

\[
N_i = \frac{1}{4} (1 - \xi)(1 - \eta)(\xi - \eta - 1); \quad N_i = \frac{1}{4} (1 + \xi)(1 - \eta)(\xi - \eta - 1); \quad N_i = \frac{1}{4} (1 + \xi)(1 + \eta)(\xi + \eta - 1); \quad N_i = \frac{1}{4} (1 - \xi)(1 + \eta)(\xi + \eta - 1); \quad N_i = \frac{1}{4} (1 - \xi)(1 - \eta)(1 - \xi); \quad N_i = \frac{1}{4} (1 + \xi)(1 - \eta)(1 - \xi); \quad N_i = \frac{1}{4} (1 + \xi)(1 + \eta)(1 - \xi); \quad N_i = \frac{1}{4} (1 - \xi)(1 + \eta)(1 - \xi);
\]

(22)

Fig 3. Doubly curved isoparametric eight noded shell element.

Here \( \xi \) and \( \eta \) are local coordinates of the element. For the isoparametric element concept, the element geometry, displacement vectors over each element are represented by the interpolation functions.

\[
x = \sum_{i=1}^{8} N_i x_i \;
\]

\[
y = \sum_{i=1}^{8} N_i y_i \; \{d\} = \sum_{i=1}^{8} N_i \{d_i\}
\]

(23)

Where, \( \{d\} = [u_0 \quad v_0 \quad w_0 \quad \theta_x \quad \theta_y ]^T \) \( \{d_i\} \) is the interpolation function for \( i^{th} \) node, \( x_i, y_i \), are geometry coordinates in \( x \) and \( y \) direction, \( \{d_i\} \) are unknown displacements vector for \( i^{th} \) node of the element. From the above equation, the displacement field is rewritten in the matrix form as

\[
\{d\} = [N]\{d_i\}
\]

(24)

Where

\[
\{d\} = [u \quad v \quad w \quad \theta_x \quad \theta_y ]^T
\]

\[
\{d_i\} = [u_i \quad v_i \quad w_i \quad \theta_{xi} \quad \theta_{yi} ]^T \; i = 1, 2, \ldots, 8
\]

(25)

(26)

(27)

Generalized strain vectors \( \{\varepsilon\} = \{\varepsilon_L\} + \{\varepsilon_{NL}\} \) at any point within the element are obtained by

\[
\{\varepsilon\} = ([B_L] + \frac{1}{2}[B_{NL}]) \{\varepsilon\}
\]

(28)

Where, \( [B_L] \) and \( [B_{NL}] \) are linear and nonlinear strain displacements matrices respectively. Where

\[
[B_L] = [B_{L1} \quad B_{L2} \quad \ldots \quad B_{L8}]
\]

(29)

and

\[
[B_{NL}] = [A] \quad [G]
\]

(30)

(31)

(32)

(33)

(34)
The of the shape functions $N_i$ with respect to global coordinates $(x$ and $y)$ are expressed in terms of their derivate with respect to $\xi$ and $\eta$ by the following relationship

$$
\left\{ \frac{\partial N_i}{\partial x}, \frac{\partial N_i}{\partial y} \right\} = [J]^{-1} \left\{ \frac{\partial N_i}{\partial \xi}, \frac{\partial N_i}{\partial \eta} \right\}
$$

(35)

Where $[J]$ is Jacobian matrix given by

$$
[J] = \begin{bmatrix}
\frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\
\frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta}
\end{bmatrix}
$$

(36)

Substituting the values of $\{E\}$ from eq. (28) in the eq. (21)

$$
\delta_{N} = \frac{1}{2} \sum \left( \{R \} \{B \} \{R \} \{B \} \right) \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta}
$$

(37)

The resulting nonlinear finite element equations are written as

$$
[K_{i} + K_{el}] \{d\} = \{F_{ext}\}
$$

(38)

In the above equation $[K_{i}]$ and $[K_{el}]$ are linear and nonlinear elastic stiffness matrix respectively, $\{F_{ext}\}$, applied mechanical load vector.

In general the above equation can be written as

$$
[K_{i}(d)] \{d\} = \{F_{ext}\}
$$

(39)

Where

$$
[K_{i}(d)] = [K_{i}] + [K_{el}]
$$

The nonlinear term $[K_{el}](d)$ in the above finite element equation depends on unknown solution $\{d\}$, thus it cannot be solved directly. So, the above equation need to be solved iteratively till the solution convergence is achieved with the said tolerance.

For geometrically nonlinear bending analysis of smart structure, the governing equation excludes the inertia and damping terms, the load becomes independent of time and the equilibrium eq.

$$
[K_{i}(d)] \{d\} = \{F_{ext}\}
$$

(40)

The above nonlinear finite element equation is linearized by using Newton-Raphson iterative method and resulting equation is given as

$$
[K_{i}]\{d\}' + \{F_{ext}\}' = \{F_{ext}\} - [K_{i}]\{d\}
$$

(41)

Where $[K_{i}]$ tangent stiffness matrix and $[K_{el}]$ secant stiffness matrix given by

$$
[K_{i}] = [K_{i} + K_{el}]
$$

(42)

$$
[K_{el}] = [K_{i}]
$$

(43)

$$
[K_{el}]_{N} = \frac{1}{2} \sum \left( \{B \} \{D \} \{B \} \{D \} \right) \{B \} \frac{\partial N_i}{\partial \xi} \{B \} \frac{\partial N_i}{\partial \eta} \{B \} \frac{\partial N_i}{\partial \xi} \{B \} \frac{\partial N_i}{\partial \eta} \{B \} 
$$

(44)

$$
[K_{el}]_{S} = \frac{1}{2} \sum \left( \{B \} \{D \} \{B \} \{D \} \right) \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta}
$$

(45)

$$
[K_{el}]_{V} = \frac{1}{4} \sum \left( \{B \} \{D \} \{B \} \{D \} \right) \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta}
$$

(46)

$$
[K_{el}]_{E} = \frac{1}{2} \sum \left( \{B \} \{D \} \{B \} \{D \} \right) \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta} \frac{\partial N_i}{\partial \xi} \frac{\partial N_i}{\partial \eta}
$$

(47)

$$
[F_{ext}] = \left\{ \begin{bmatrix}
\frac{\partial N_i}{\partial \xi} \\
\frac{\partial N_i}{\partial \eta}
\end{bmatrix} \right\} \{q_\alpha\}
$$

(48)

Where $\{q_\alpha\}$ is surface traction mechanical load.

### VI. RESULTS AND DISCUSSION

By using the above described analysis procedure a nonlinear finite element code is developed in Ansys 11. In order to demonstrate the accuracy and applicability of the present formulation, several examples have been analyzed and the computed results are compared with published results. In all problems it is considered that all individual layers are taken to be equal thickness. In the given examples the deflection and the load are non-dimensionalized as follows:

Central deflection:  $\overline{W} = \frac{W}{h}$

Load: uniformly distributed load $Q = qa^4/E_t^4$

Where $a$ is plate dimension; $q$ is uniformly distributed load; $t$ is thickness of plate. The present boundary condition are taken in the analysis.

All edges simply supported (SS):

- $u = v = w = 0$, along $y$ axis
- $u = w = 0$, along $x$ axis

All edges clamped (CC):

- $u = v = w = 0$, along $x$ and $y$ axis

All edges hinged (HH):

- $u = v = w = 0$, along $x$ axis
- $u = w = 0$, along $y$ axis

### VII. VALIDATION STUDY

Present geometrically nonlinear formulation is validated by comparing the author results and existing literature for cross ply plates. The finite element results are compared with experimental results of Zaghoul and Kennedy [13]. The center deflection of all edges clamped symmetric bi-directional square plate under uniform loading is presented in Fig. 4. The clamped boundary conditions are used for the plate. The material properties used for computation are:

- $E_x = 1.8315 \times 10^6$, $E_y = 1.8282 \times 10^5$, $G_{xy} = G_{yx} = 3.125 \times 10^4$, $\vartheta = 0.02349490$

The comparisons shows that the present results and experimental results are in good agreement.

![Fig 4 Comparison of the present FEM results with experiments results for a simply supported 4-ply symmetric plate](image-url)
Numerical examples:
In this section numerical analysis is carried out by considering different boundary conditions to the know the geometric nonlinear behaviour of anti-symmetric cross ply laminated plates. In the present analysis the effect of number of layers and the lamination scheme on the center deflection is examined for anti-symmetric cross-ply laminates. A square plate with clamped, simply supported, hinged boundary condition and material. The material properties of the laminates are taken as follows:

\[ E_2 = 1.45 \times 10^5, \quad E_1 = 3.63 \times 10^6, \quad G_{12} = 0.5 \times E_2, \quad \theta = 0.3 \]

From Fig. (5-7) it can be concluded that as the number of layers is increases, the center deflection decreases and number of layers reduces the nonlinearity in the cross-ply laminates.

![Fig.5 Effects on number of layers on transverse displacement of clamped anti-symmetric cross-ply plate](image1)

![Fig.6 Effects on number of layers on transverse displacement of hinged anti-symmetric cross-ply plate](image2)

![Fig.7 Effects on number of layers on transverse displacement of simply supported anti-symmetric cross-ply plate](image3)

In Fig.8 it clear that plate with clamped boundary condition and gives lower displacement when compared to the simply supported boundary condition. In clamped plates, all essential/natural boundary conditions are satisfied but in the simply supported laminated plate with immovable edges, moment free condition are not satisfied.

CONCLUSION

In this paper, the geometric nonlinear analysis of plates are carried out by using a Newton Raphson method. The validation study is carried out and validation results are good in terms of experimental results. Numerical results are obtained for nonlinear center deflections of square plates subjected to various edge conditions. It is concluded that the number of layers, stacking sequence, boundary condition have significant effect on the nonlinear deflection of the plate.

REFERENCE


