

DESIGN OF WHITWORTH QUICK RETURN MECHANISM USING NON-CIRCULAR GEARS

¹JASTI SAI PRANEETH, ²ANIL KUMAR SREEDHARALA, ³NAVEEN KUMAR DARISI, ⁴M ELANGO VAN

¹Student, ²Student, ³Student, ⁴Professor

Abstract- In this paper, a mechanism similar in function to Whitworth quick return mechanism is designed using a pair of non-circular gears integrated into Scotch-yoke mechanism. The velocity profile of the slider in quick return mechanism is derived analytically and used as reference to the velocity of slider in proposed design.

Index Terms- Elliptical gears, non-circular gears, Scotch-yoke mechanism, Whitworth quick return mechanism

I. INTRODUCTION

Circular gears are widely used in design of wide range of mechanisms. There are no practical limitations to the usage of gears but their versatilities can be maximised when used as non-circular gears. Non-Circular gears are gears with distorted shapes that are available mostly as elliptical gears, eccentric circular gears, oval gears, or internal noncircular gears and of higher order. Non-Circular gears are used to improve the function, versatility, and simplicity of many mechanical processes.

Noncircular gears have been considered a curiosity and a product of niche applications for a long time because of their design and manufacture complexity. Our paper tries to contemplate a better inside by designing an application based on non-circular gears.

Whitworth Quick-Return Mechanism is second inversion of the slider crank mechanism attained by fixing the crank instead of the ground link. The peculiar feature of this mechanism is that its forward stroke takes a little longer and cuts the metal whereas the return stroke takes a shorter period.

The shaper machine used in lathe workshops works on the principle of Whitworth quick return mechanism (as shown in Fig 1.1). Quick return mechanism consists of many linkages that rotates at high speeds.

The linkages tend to have dynamic effects. Balancing weight are to be added to these linkages to avoid vibrations. This intern increases the overall weight of the mechanism.

The proposed mechanism intended to replace the conventional mechanism with a compact, robust design.

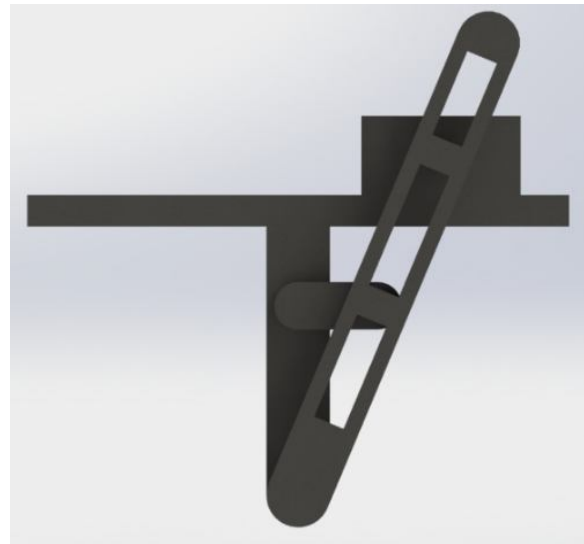


Fig1.1: Whitworth Quick Return Mechanism

The velocity profile of the slider in quick return mechanism is shown in Fig 1.2. This paper tries to recreate the features of the profile.

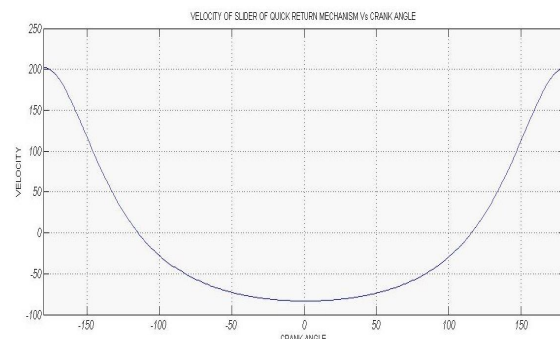


Fig 1.2: Velocity profile of slider of Whitworth quick return mechanism using MATLAB

The scotch yoke mechanism is a very simple mechanism for converting rotary motion into reciprocating motion. This mechanism is basically a second inversion of double slider crank mechanism

obtained when any of the sliding blocks is fixed. The velocity profile of the slider in scotch-yoke mechanism is sinusoidal in nature as shown in Fig 1.3. This profile is to be imposed by another function that is

created by a pair on non-circular gears in order to create a velocity profile similar to that of quick return mechanism as shown in Fig 1.2

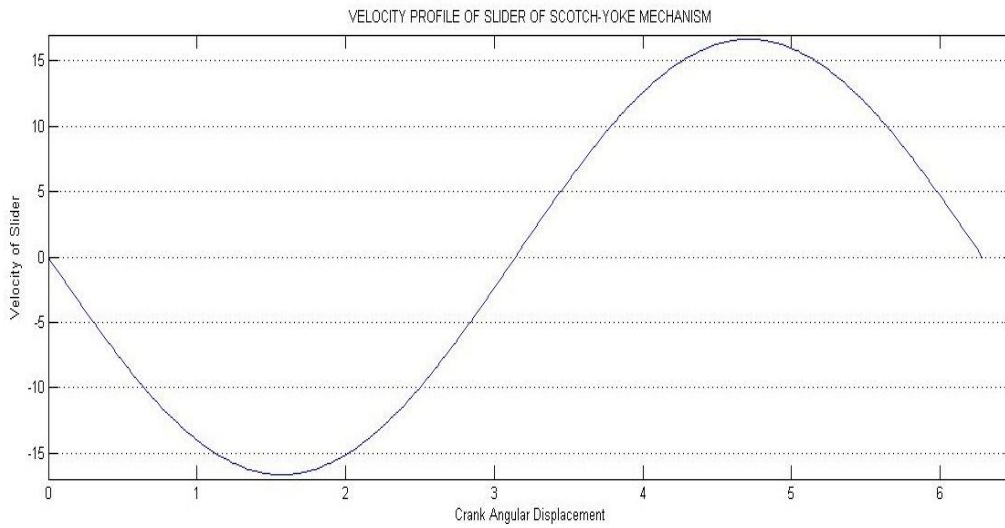


Fig 1.3: Velocity profile of slider in Scotch-yoke mechanism

II. MODIFIED SCOTCH-YOKE MECHANISM:

The Scotch-Yoke mechanism in the proposed design is driven by a pair of non-circular gears 1 and 2 and two circular gears 3 and 4 (Fig. 2.1(a)) so that the

input to the conventional scotch-yoke mechanism is no longer a constant i.e., $\omega_5 \neq \text{constant}$ (Fig 2.1(b)).

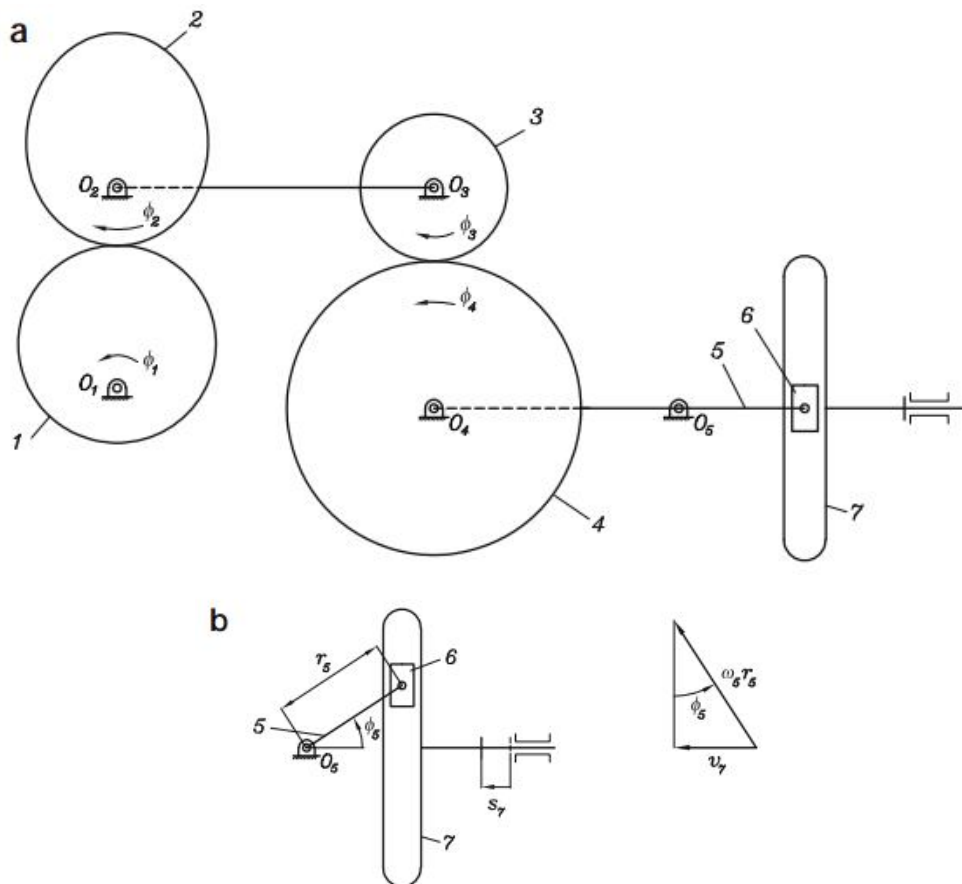


Fig 9.2.1: Composition of gear drive with scotch-yoke mechanism: (a) eccentric gear drive and scotch-yoke mechanism at their initial positions and (b) links of scotch yoke mechanism and details of related velocities

The purpose of this design is to improve the function of output velocity $v_7(\varphi_1)$ of the Scotch-Yoke mechanism (coupled with gears) with the following conditions.

1. Gear 1 (Fig. 2.1(a)) is an elliptical involute gear and noncircular gear 2 is conjugated to gear 1.
2. Gears 3 and 4 (Fig. 2.1(a)) are circular gears with gear ratio of 1 ($\frac{\omega_{(3)}}{\omega_{(4)}} = 1$); gear 4 performs one revolution for one revolutions of gear 3.
3. Function $v_7(\varphi_1)$ is of a period $\varphi_1 = 2\pi$, and this is obtained by application of circular gears 3 and 4 in addition to elliptical gear 1 and noncircular gear 2.

The basic kinematic relation of the coupled mechanism is based on the drawings of Figure 2.1(a) shows that the initial position of links at $\varphi_1 = 0$.

The basic kinematic relations of the velocities of the links of the mechanism are represented by the following equations

$$v_7 = \omega_E r_E \sin \varphi_E \tag{2.1}$$

Whereas φ_E is the angle of rotation of the crank. Such a relation is a sine type function.

Angular velocity ω_E is determined as

$$\omega_E = \omega_4 = m_{43} \omega_3 = m_{43} \omega_2 = m_{43} m_{21}(\varphi_1) \omega_1 \tag{2.2}$$

Angle of rotation φ_E is also derived as

$$\varphi_E = \varphi_4 = m_{43} \varphi_3 = m_{43} \varphi_2 = m_{43} \left[\int_0^{2\pi} m_{21} d\varphi_1 \right] \tag{2.4}$$

Fig.9.2.2 shows the velocity function for different values of parameter of eccentricity e_1 when a tandem design based on one-pair of elliptical gears and a Scotch-Yoke mechanism is applied.

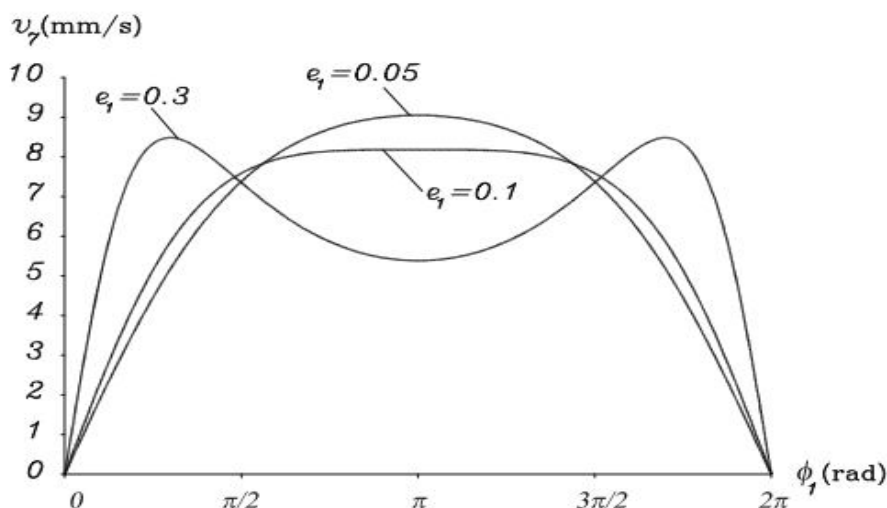


Fig 9.2.2: Velocity of the scotch-yoke as a function of angle of rotation of an elliptical gear for different values of the parameter of eccentricity e_1

III. DESIGN OF ELLIPTICAL GEARS

In figure 3.1, the noncircular gears (elliptical gears) 1 and 2 with angular velocity ω_1 and ω_2 and angular

displacement φ_1 and φ_2 , respectively, reversely rotate around the fixed axes O_1 and O_2 . Coordinate systems X_1Y_1 and X_2Y_2 are the moving coordinate attached to gears 1 and 2, respectively.

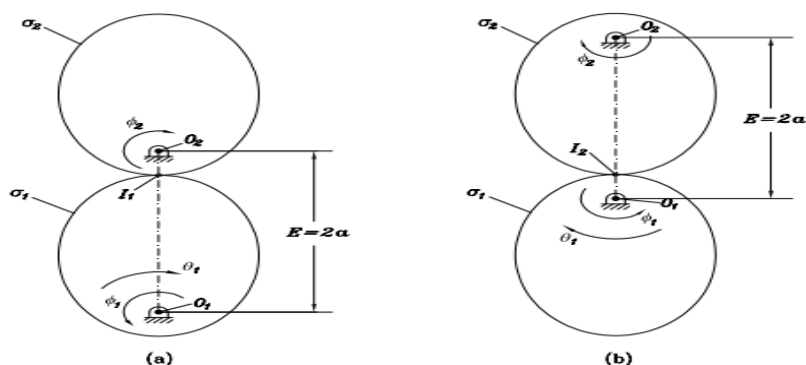


Fig 3.1: Illustration of two initial positions of gear 1 and 2 where O_1 is (a) the lower focus of ellipse 1; (b) the upper focus of ellipse 1

The driving centrode is represented in polar form by

$$r_1(\phi_1), \quad 0 \leq \phi_1 \leq 2\pi n \quad (3.1)$$

The rotating- speed ratio or transmission ratio $m_{21}(\phi_1)$ is:

$$m_{21}(\phi_1) = \frac{d\phi_2}{d\phi_1} = \frac{r_2(\phi_2)}{r_1(\phi_1)} = \frac{r_2(\phi_2)}{E - r_1(\phi_1)} \quad (3.2)$$

Where $E = r_1(\phi_1) + r_2(\phi_1)$ is the center distance. Similarly, derivative function $m_{21}(\phi_1)$ is represented by

$$m_{12}(\phi_1) = \frac{d\phi_1}{d\phi_2} = \frac{r_2(\phi_2)}{r_1(\phi_1)} = \frac{E - r_1(\phi_1)}{r_1(\phi_1)} \quad (3.3)$$

The meaning of Eqn. (3.2) and (3.3) is that centrodes σ_1 and σ_2 roll over each other. Function $m_{12}(\phi_1)$ must be a periodic one to obtain a closed form curve

$$m_{21}(\phi_1) = \frac{\omega_2}{\omega_1} = \frac{\frac{d\phi_2}{dt}}{\frac{d\phi_1}{dt}} = \frac{d\phi_2}{d\phi_1} \quad (3.4)$$

When $\phi_1 = 0$ then ϕ_2 also be zero.

Integrating equation (3.3.1) with respect to ϕ_1 , the relationship of ϕ_1 and ϕ_2 is obtained:

$$\phi_2(\phi_1) = \frac{2\pi}{n} = \int_0^{2\pi} m_{21} d\phi_1 = \int_0^{2\pi} \frac{r_2(\phi_2)}{E - r_1(\phi_1)} d\phi_1 \quad (3.5)$$

Where n is an integer number and represents the number of revolutions of the driving centrode for one revolution of the driven centrode.

Eqn. 3.3 can be interchanged so that the centrode of elliptical gears can be defined as

$$r_1(\phi_1) = \frac{E + m_{21}(\phi_1)}{1 + m_{21}(\phi_1)} \quad \& \quad (3.6)$$

$$r_2(\phi_2) = \frac{E}{1 + m_{21}(\phi_1)} \quad (3.7)$$

A conventional elliptical gear drive is formed by two elliptical gears that may be assembled as shown in Figs. 3.1(a) or 3.1(b). Figure 3.1(a) shows a gear drive wherein centre O_1 of rotation of centrode σ_1 coincides with the lower focus F of ellipse 1. On the other side, Figure 3.1(b) shows a gear drive wherein centre O_1 of rotation of centrode σ_1 coincides with the upper focus F^* of ellipse 1.

Figure 3.2 shows an elliptical centrode that performs rotation around centre O_1 (it is the lower focus F of ellipse 1). We recall that the instantaneous point I of tangency of centrode 1 and centrode 2 (centrode 2 is not shown) moves along centre distance O_1O_2 (Fig. 3.2) in the process of motion. The centre distance is $E = 2a$, where $2a$ is the major axis of mating elliptical centrodes 1 and 2. Centrode σ_1 (Figs. 3.1(a) and 3.2) is represented by

$$r_1(\phi_1) = \frac{p}{1 - e \cos \phi_1} = \frac{a(1 - e^2)}{1 - e \cos \phi_1} \quad (3.8)$$

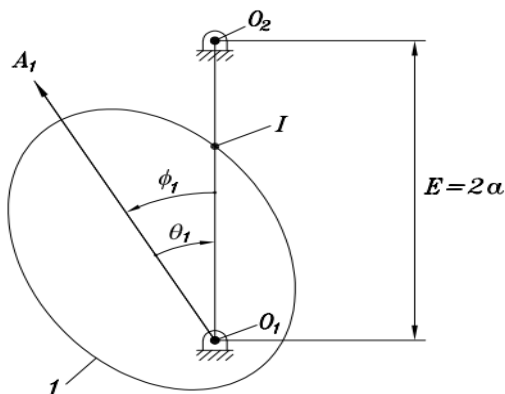


Fig 3.2: For derivation of transmission function $\phi_2(\phi_1)$ of elliptical gears

The derivative function $m_{12}(\phi_1)$ is derived to

$$m_{12}(\phi_1) = \frac{d\phi_1}{d\phi_2} = \frac{r_2(\phi_2)}{r_1(\phi_1)} = \frac{E - r_1(\phi_1)}{r_1(\phi_1)} \quad (3.9)$$

where $r_1(\phi_1)$ determines the polar equation of centrode. Taking into account Eq.(3.8), $E = 2a$, and $p = a(1 - e^2)$, we obtain,

$$m_{12}(\phi_1) = \frac{1 + e^2 - 2e \cos(\phi_1)}{1 - e^2} \quad (3.10)$$

The sought-for transmission function may then be represented as

$$\phi_2(\phi_1) = \int_0^{\phi_1} \frac{d\phi_1}{m_{12}(\phi_1)} = (1 - e^2) \int_0^{\phi_1} \frac{d\phi_1}{1 + e^2 - 2e \cos(\phi_1)} \quad (3.11)$$

We obtain transmission function $\phi_2(\phi_1)$ as a closed form solution as follows

$$\tan \frac{\phi_2}{2} = \frac{1 + e}{1 - e} \tan \frac{\phi_1}{2} \quad (3.12)$$

The transmission function represented in Eq. (3.12) corresponds to the assembly positions of centrodes as shown in Fig. 3.1(a). For assembly as shown in Fig. 3.1(b), we obtain

$$\tan \frac{\phi_2}{2} = \frac{1-e}{1+e} \tan \frac{\phi_1}{2} \quad (3.13)$$

IV. PROPOSED DESIGN

From the Eqn. 2.2 we derived the angular velocity ω_5 as

$$\omega_5 = \omega_4 = m_{43}\omega_3 = m_{43}\omega_2 = m_{43}m_{21}(\phi_1)\omega_1 \quad (2.2)$$

Wherein $m_{21}(\phi_1)$ is derived earlier as

$$m_{21} = \frac{1-e^2}{1+e^2-2e\cos\phi_1} \quad (4.1)$$

$$\text{Hence, } \omega_5 = m_{43}\omega_1 \left[\frac{1-e^2}{1+e^2-2e\cos\phi_1} \right] \quad (4.2)$$

Where e is the eccentricity of the ellipse.

Angle of rotation ϕ_5 is also derived as

$$\phi_5 = \phi_4 = m_{43}\phi_3 = m_{43}\phi_2 = m_{43} \left[\int_0^{2\pi} m_{21} d\phi_1 \right] \quad (4.3)$$

Eqn. (2.1) – (7.2.4) yield

$$v_7 = m_{43}m_{21}\omega_1 r_5 \sin \left(m_{43} \left[\int_0^{2\pi} m_{21} d\phi_1 \right] \right) \quad (4.4)$$

Eqs. (2.1), (4.2), (4.3), (4.4) yield

$$v_7 = m_{43}m_{21}\omega_1 r_5 \sin \left(m_{43} \left[\int_0^{2\pi} m_{21} d\phi_1 \right] \right) \quad (4.5)$$

A matlab code was generated to design the pair of elliptical gears based on the mathematical equations derived earlier. Refer Appendix-I for code to generate centre of elliptical gears.

The Cartesian coordinates of these plots are imported into Solidworks through the module, curve through XYZ. Once they were imported into Solidworks, tooth profile needs to be generated over the centrodes. For this a circle with radius P_{\min} is drawn tangent to the centre as shown in Fig 3.3

$$\text{Where, } P_{\min} = \frac{b^2}{a} = a(1-e^2) \quad (4.6)$$

The number of teeth of an elliptical gear is determined by

$$N = \frac{4a}{m\pi} \int_0^{\pi} \sqrt{1-e^2 \sin^2 \psi} d\psi \quad (4.7)$$

Once the tooth is drawn it is arrayed on the surface not along the circle but along the curve imported earlier. On completion of tooth profile the elliptical gear design is completed. The other conjugate elliptical gear is also designed in the same procedure.

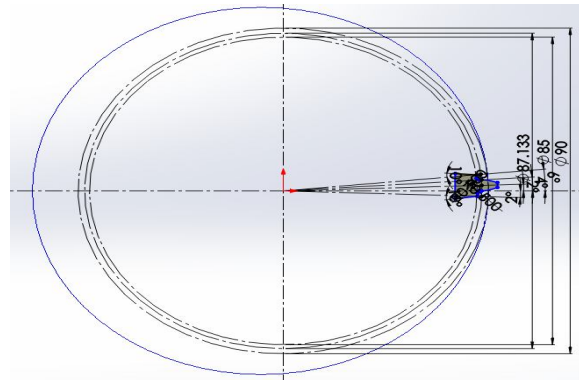
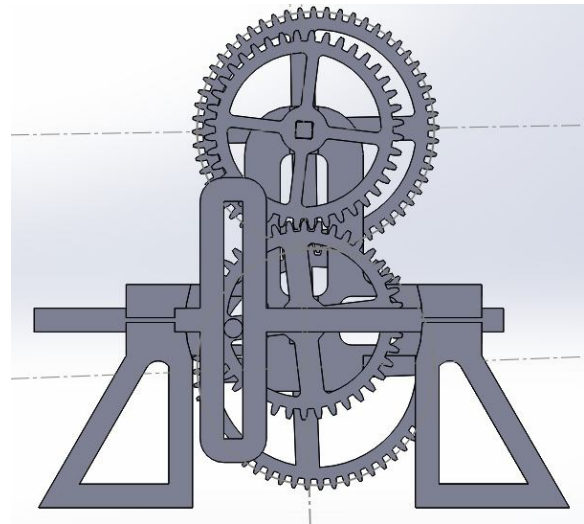


Fig3.3: Tooth profile generated onto the centre of elliptical gear

A prototype model of Scotch-yoke mechanism integrated with noncircular elliptical gears is designed as shown in Fig. 3.4



CONCLUSION

The velocity profile of the slider is plotted under given conditions and parameters using MATLAB as shown in Fig. 11.1

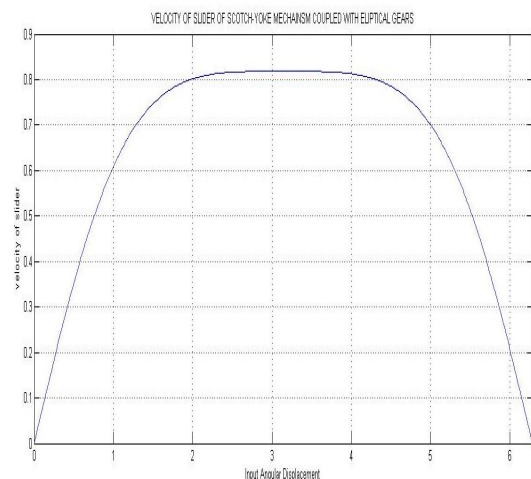


Fig 11.1: Velocity profile of slider of Scotch-yoke mechanism coupled with elliptical gears

On careful inspection of the profile it is evident that while return stroke i.e., when crank angular displacement increases from 0^0 to 90^0 , the velocity of slider increase gradually and then dwells at the maximum velocity during entire return stroke.

During forward stroke the velocity of slider drops gradually to zero by the time the slider is at full stroke. This evidently proves that the elliptical gears are designed correctly and an alternate mechanism to that of quick return mechanism is designed successfully.

REFERENCES

- [1]. Bair, B.W., 2002, "Computer Aided Design of Elliptical Gears," Transactions of the ASME, Journal of Mechanical Design, Vol.124, pp.787-793.
- [2]. Bair, B.W., 2004, "Computer Aided Design of Elliptical Gears with Circular-arc Teeth," Mechanism and Machine Theory, Vol.39, No.2, pp.153-168.
- [3]. Bair, B.W., "Computer aided design of non-standard elliptical gear drives," Proceedings of the Institution of Mechanical Engineers Part C: Journal of Mechanical Engineering Science, 2002, 216(4):473-483.
- [4]. Danieli, G.A., 2000, "Analytical Description of Meshing of Constant Pressure Angle Teeth Profile on Variable Radius Gear and its Applications" Transactions of the ASME, Journal of Mechanical Design, Vol.122, pp.123-129.
- [5]. Faydor L. Litvin, Ignacio Gonzalez-Perez, Alfonso Fuentes, Kenichi Hayasaka," Noncircular Gears: Design and Generation," Cambridge university press, United states of America, 2009
- [6]. G.A. Danieli, D. Mundo, "New developments in variable radius gears using constant pressure angle teeth profiles," Mechanism and Machine Theory 40 (2005) 203–217.
- [7]. Liu, Jen-Yu and Chen, Yen-Chuan, "A design for the pitch curve of noncircular gears with function generation", "Proceedings of the International Multi Conference of Engineers and Computer Scientists", Vol II, 2008.
- [8]. Ottaviano, Erika, et al., "Numerical and experimental analysis of non-circular gears and cam-follower systems as function generators", "Mechanism and machine theory", Vol. 43, 996-1008, 2008.
- [9]. S.L. Chang, C.B. Tsay, "Computerized tooth profile generation and undercut analysis of noncircular gears manufactured with shaper cutters," Journal of Mechanical Design, ASME Transactions 120 (1998) 92–99.
- [10]. X. Wu, S. Wang, A. Yang, "Non-circular gear CAD/CAM technology, in: Proceedings of the Eighth World Congress on the Theory of Machines and Mechanisms, Prague, Czechoslovakia, 1991, pp. 391–394.
- [11]. Yongping Liu, Pengfei Meng," Parametric design of the pitch curves of higher-order elliptic gears," Science Technology and Engineering, 2010, 10(1): 39-42.
- [12]. Vasiu Marius, Andrei Laurenția," Technologies for Non-Circular Gear Generation and Manufacture", The Annals "Dunărea De Jos" Of Galați Fascicle V, Technologies in Machine Building, ISSN 1221-4566, 2010.

APPEDIX I MATLB CODE FOR GENERATION OF CENTRODE OF A PAIR OF ELLIPTICAL GEARS

```
% assigning variables %
>> e = .1;

% transmission ratio of circular gears%
>> y = 1;

% Centre distance between elliptical gears%
>> a = 50;
>> x = (0 : sym ('1/360'): 1);
>> theta = 2*pi*x;
>> for n = 1:361

% defining the transmission ratio of elliptical gears%
m(n) = (1+(e^2)-2*e.*cos(theta(n)))/(1-e^2);

% Defining the centrodes of both gears%
r1(n) = a*(1-e^2)/(1-e*cos(theta(n)));
r2(n) = 2*a - r1(n);

%Deriving the phase change between the gears%
theta2(n) = theta(n) + 2*atan((1+tan(theta(n)/2))/9) -
2*atan(tan(theta(n)/2));
angle(n) = atan(((1+e)/(1-e)*tan(theta(n)/2)));
end

% Rectifying the errors of phase%
>> for n = 1:361

if n>=1 & n <=181
sample(n) = sin(y*angle(n));
end

if n>181
sample(n) = -sin(y*angle(n));
end
end

% plotting the centrodes of both gears%
>> plot(r1.*sin(theta),r1.*cos(theta))
>> plot(r2.*sin(theta),r2.*cos(theta))

%Plotting the velocity profile of output slider%
>> plot(theta,sample./m)
```

